B.Sc. Part-I (Sub) Set Theory 08-07-2020

Q.1. Prove that

$$(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$$

Soln. Let $(x,y)\in(A\times B)\cap(C\times D)$ be arbitrary, Then

$(x,y)\in(A\times B)\cap(C\times D)$

$\Rightarrow (x,y)\in(A\times B)\wedge(x,y)\in(C\times D)$

$\Rightarrow (x\in A\wedge y\in B)\wedge(x\in C\wedge y\in D)$

$\Rightarrow (x\in A\wedge x\in C)\wedge(y\in B\wedge y\in D)$

$\Rightarrow x\in(A\cap C)\wedge y\in(B\cap D)$

$\Rightarrow (x,y)\in(A\cap C)\times(B\cap D)$

$\therefore (A\times B)\cap(C\times D)\subseteq(A\cap C)\times(B\cap D)$ —— (1)

Again, Let $(a,b)\in(A\cap C)\times(B\cap D)$ be arbitrary.

Then, $(a,b)\in(A\cap C)\times(B\cap D)$

$\Rightarrow a\in(A\cap C)\wedge b\in(B\cap D)$

$\Rightarrow (a\in A\wedge a\in C)\wedge(b\in B\wedge b\in D)$

$\Rightarrow (a\in A\wedge b\in B)\wedge(a\in C\wedge b\in D)$

$\Rightarrow (a,b)\in(A\times B)\wedge(a,b)\in(C\times D)$

$\Rightarrow (a,b)\in(A\times B)\cap(C\times D)$

$\therefore (A\cap C)\times(B\cap D)\subseteq(A\times B)\cap(C\times D)$ —— (2)

Now, from (1) and (2)

$$(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$$

Proved.

Q.2. Prove that

$$(A-B)\times C=(A\times C)-(B\times C)$$

Soln. Let $(x,y)\in(A-B)\times C$ be arbitrary. Then

$(x,y)\in(A-B)\times C$

$\Rightarrow x\in(A-B)\wedge y\in C$

$\Rightarrow (x\in A\wedge x\notin B)\wedge y\in C$

$\Rightarrow (x\in A\wedge y\in C)\wedge(x\notin B\wedge y\in C)$

$\Rightarrow (x,y)\in(A\times C)\wedge(x,y)\notin(B\times C)$

B.Sc. Part-I (Sub) Set Theory. 08-07-2020

based on Cartesian products Theorem

Rest part of Q2.

$\Rightarrow (x,y) \in (A\times C) - (B\times C)$

$\therefore (A-B)\times C \subseteq (A\times C) - (B\times C)$

Again let $(a,b) \in (A\times C) - (B\times C)$ be arbitrary. Then

$(a,b) \in (A\times C) - (B\times C)$

$\Rightarrow (a,b) \in (A\times C) \wedge (a,b) \notin (B\times C)$

$\Rightarrow (a\in A \wedge b\in C) \wedge a\notin B$

$\Rightarrow (a\in A \wedge a\notin B) \wedge b\in C$

$\Rightarrow a\in (A-B) \wedge b\in C$

$\Rightarrow (a,b) \in (A-B)\times C$ —— (2)

$\therefore (A\times C) - (B\times C) \subseteq (A-B)\times C$

Now, from (1) and (2),

$(A-B)\times C = (A\times C) - (B\times C)$

Q.3. If A, B, C be three sets, $A \neq \phi$, $B \neq \phi$ and $(A\times B) \cup (B\times A) = C\times C$

Prove that $A = B = C$

Soln: It is given that $A \neq \phi$, $B \neq \phi$

$\Rightarrow A\times B \neq \phi$, $B\times A \neq \phi$

$\Rightarrow (A\times B) \cup (B\times A) \neq \phi$

$\Rightarrow C\times C \neq \phi$ $[\because (A\times B)\cup(B\times A) = C\times C]$

$\Rightarrow C \neq \phi$.

Now, $x\in A$, $y\in B \Rightarrow (x,y) \in (A\times B)$

$\Rightarrow (x,y) \in (A\times B) \cup (B\times A)$

$\Rightarrow (x,y) \in C\times C$

$\Rightarrow x\in C$, $y\in C$ —— (1)

$\therefore A\subseteq C$, $B\subseteq C$

Again, $a\in C \Rightarrow (a,a) \in C\times C$

$\Rightarrow (a,a) \in (A\times B) \cup (B\times A)$

$\Rightarrow a\in A$, $a\in B$ —— (2)

$\therefore C\subseteq A$, $C\subseteq B$

From (1) and (2), we have

$A = B = C$. Proved.